



The Thickness Effect of the Degenerate Plasma Layer on the Dispersion Relation of Cylindrical Smooth-wall Waveguide

Fatemeh Asadiamiri, Ali Kashif Chaudhary, Mahdi Bahadoran, Jalil Ali
Institute of Advanced Photonics Science,
Universiti Teknologi Malaysia (UTM)
81310 Johor Bahru, Malaysia

*fatemeh.asadiamiri@yahoo.com, kashif@utm.my, bahadoran@utm.my, jalilali@utm.my

Mehdi Baboli
Dept. of Electronics and Computer Engr.
Faculty of Electrical Engineering,
Universiti Teknologi Malaysia (UTM)81310 Johor Bahru, Malaysia
mehdi.baboli@fkegraduate.utm.my

Malihe Nejati
Physics Department, Qom University
Qom University
malihe_nejati@yahoo.com

Preecha Yupapin
King Mongkut's Institute of Technology Ladkrabang (KMITL),
Bangkok 10520 Thailand
kypreech@kmitl.ac.th

Abstract— In this paper dispersion relation in TE mode in cylindrical waveguide contain dielectric rod and plasma layer are investigated using Maxwell equations, and boundary conditions. The dispersion relation is simulated in slow wave. The effect of plasma radius variation on the frequency spectra of slow waves are simulated by maple software. It is found that decrease in plasma radius causes a higher frequency in slow waves which is practical in telecommunication. Furthermore, the frequency spectrum of the waves is slightly shifted by thickness of the umagnetized degenerate plasma layer.

Keywords — Dispersion relation, cylindrical waveguide, dielectric rod, plasma layer, Maxwell equation

I. INTRODUCTION (*HEADING 1*)

Recently there is an increasing interest in coherent radiation form short bunches of electrons as a source of millimeter and sub-millimeter waves [1]. It is well known that microwave energy is widely used to heat and drive steady-state currents in magnetically confined plasmas for

controlled fusion experiment [2]. Electrons and ions in plasma are natural species in that become a subject to numerous natural oscillations particularly in the microwave frequency range. For this reason, a device filled with plasma may serve as a source of microwave radiation [1,3]. Moreover, the electron beam-plasma interaction have been used as a high power source of microwave and millimetre radiation [4]. However, it is released that the injection of background plasma in the electron-wave interaction region can reduce the space-charge wave limitations [5]. Furthermore it has been found experimentally that the injection of plasma into the microwave apparatus enhances the interaction efficiency and the output power much more than that of for vacuum case. Plasma also can improve the transmission quality of electron beam as light beam transmit without a guiding magnetic field. Thus high-power microwave devices get lots of attention [6].

In this paper the electromagnetic dispersion relation in TE mode in cylindrical waveguide with dielectric rod and unmagnetized degenerate plasma layer are investigated. The effect of plasma radius variation on the slow waves frequency are simulated by maple software. We found that for plasma with smaller radius, a higher frequency in slow waves will produce. This attribute of plasma can be used for telecommunication applications. Additionally, the frequency spectrum of the waves is slightly shifted by thickness of the unmagnetized degenerate plasma layer.

II. THEORY

The structure of cylindrical waveguide with radius R_m for unmagnetized degenerate plasma layer with radius R_p which covered dielectric rod with radius R_d in center of waveguide is shown in Figure 1.

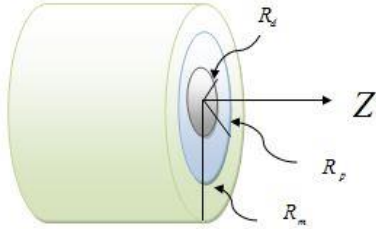


Figure 1: The cylindrical waveguide structure with dielectric rod and unmagnetized degenerate plasma layer

To study the dispersion relations of electromagnetic waves in proposed structure, Maxwell's equations are used [3]

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial}{\partial t} (\varepsilon \vec{E}) \quad (1)$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial}{\partial t} \vec{B} \quad (2)$$

where \vec{B} and \vec{E} represents the perturbed values of magnetic and electric fields. Here ε shows the dielectric tensor. The system of fundamental equations describing general behaviour of symmetric magnetic field in this geometry can be attained as follows [3]

$$\nabla_{\perp}^2 B_z - x^2 B_z = 0 \quad (3)$$

where

$$x^2 = k_z^2 - \varepsilon_{\perp} \frac{\omega^2}{c^2} \quad \& \quad \nabla_{\perp}^2 = \frac{1}{r} \frac{d}{dr} r \frac{d}{dr} \quad (4)$$

Here k_z shows the axial wave number, ε_{\perp} represents the transverse component of plasma dielectric tensor, ω is angular frequency of electromagnetic wave, c shows the speed of light and ∇_{\perp}^2 is the transverse Laplacian. Eq. 3 describes TE symmetric wave's propagation along the axis of the waveguide with non-zero field components of axial electric field E_z , angular magnetic field B_{ϕ} and radial magnetic field B_r .

III. SLOW WAVES

To achieve the slow waves [4], the phase velocity should be smaller than the speed of light in vacuum ($v_{ph} = \omega/k_z < c$) thus the following parameters are introduced

$$\begin{cases} x'_p = \sqrt{\left(-\varepsilon_{\perp}^p \frac{\omega^2}{c^2} + k_z^2\right)}, \\ x'_d = \sqrt{\left(\varepsilon_d \frac{\omega^2}{c^2} - k_z^2\right)}, \\ x'_v = \sqrt{\left(k_z^2 - \frac{\omega^2}{c^2}\right)} \end{cases} \quad (5)$$

where in Eq. (5) ε_{\perp}^p is defined as [7]

$$\varepsilon_{\perp} = \varepsilon_{\perp}^p - 1 - \sum_{\alpha} \frac{3\omega_{p\alpha}^2}{\omega^2} \left\{ \begin{array}{l} \left(\frac{\omega}{k_z V_{F\alpha}} \right)^2 \times \\ - \frac{\omega}{2k_z V_{F\alpha}} \ln \left(\frac{\omega + k_z V_{F\alpha}}{\omega - k_z V_{F\alpha}} \right) \\ \times \left(\left(\frac{\omega}{k_z V_{F\alpha}} \right)^2 - 1 \right) \end{array} \right\} \quad (6)$$

In Eq.6 $\omega_{p\alpha}$ is the angular frequency of electromagnetic wave and the summation extends over all types of charged

particles in the degenerate plasma. $V_{F\alpha}$ is Fermi velocity of charge carriers of type α of degenerate plasma .

Based on the Eq. 3, the magnetic field for different parts of cylindrical waveguide is

$$B_z = \begin{cases} D_1 J_m(x_d r) & r < R_d \\ D_2 K_m(x'_p r) + D_3 I_m(x'_p r) & R_d < r < R_p \\ D_4 K_m(x'_v r) + D_5 I_m(x'_v r) & R_p < r < R_m \end{cases} \quad (7)$$

Where D_i ($i=1,2,..5$) are constant coefficients. J_m , I_m and K_m are representing the Bessel function of the m th order, the first kind and the second kind of modified Bessel function in m th order, respectively. In addition x'_p , x_d and x'_v are real values in limited area

$$\frac{c}{\sqrt{\epsilon_d}} < \frac{\omega}{k_z} < c \quad (8)$$

Considering the boundary conditions [2]

$$\left\{ \begin{array}{l} \{B_z\}_{r=R_d, R_p} = 0 \\ \{E_\phi\}_{r=R_d, R_p} = 0 \\ E_\phi|_{r=R_m} = 0 \end{array} \right. \quad (9)$$

the dispersion relation is obtained from the following determinant

$$\begin{vmatrix} J_m(x_d R_d) & -K_m(x'_p R_d) & -I_m(x'_p R_d) & 0 & 0 \\ 0 & -K_m(x'_p R_p) & I_m(x'_p R_p) & K_m(x'_v R_p) & -I_m(x'_v R_p) \\ 0 & 0 & 0 & K'_m(x'_v R_m) & I'_m(x'_v R_m) \\ \frac{J'_m(x_d R_d)}{x_d} & -\frac{K'_m(x'_p R_d)}{x'_p} & -\frac{I'_m(x'_p R_d)}{x'_p} & 0 & 0 \\ 0 & \frac{K'_m(x'_p R_p)}{x'_p} & \frac{I'_m(x'_p R_p)}{x'_p} & -\frac{K'_m(x'_v R_p)}{x'_v} & -\frac{I'_m(x'_v R_p)}{x'_v} \end{vmatrix} = 0 \quad (10)$$

In Figure 2 the frequency is normalized by plasma frequency ω_{pe} , and k_z is normalized by c/ω_{pe} .

Figure 2 shows the dispersion relation for two different plasma radii in slow waves with smooth-wall waveguide. Simulated results show that a decrease in plasma radii R_p causes increase in frequency of waves.

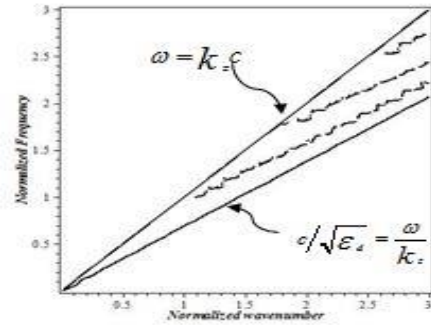


Figure 2. simulated result for dispersion relation of slow wave for two different plasma radii of $R_p = 8 \mu m$ (dash lines) and $R_p = 7.1 \mu m$ (dot lines).

IV. CONCLUSIONS

In this paper the electromagnetic dispersion relation in TE mode for dielectric rod and unmagnetized degenerate plasma layer is investigated. It is found that decrease in plasma radius causes a higher frequency in slow waves in TE mode. Also, the frequency spectrum of the waves is slightly shifted by thickness of the unmagnetized degenerate plasma layer.

ACKNOWLEDGMENT

We would like to thank the Institute of Advanced Photonics Science, Nanotechnology Research Alliance, Universiti Teknologi Malaysia (UTM) and King Mongkut's

Institute of Technology Ladkrabang (**KMITL**), Thailand for providing the research facilities. This research work has been supported by UTM's Tier 1/Flagship Research Grant and IDF financial support from UTM.

REFERENCES

- [1] B. Jazi, M. Nejati, and A. Salehi, "The theoretical investigation of THz electromagnetic waves in a rod degenerate plasma-waveguide," *International journal of infrared and millimeter waves*, vol. 27, pp. 1469-1495, 2006.
- [2] JB. Jazi, B. Shokri, and H. Arbab, "Azimuthal electromagnetic surface waves in a rod dielectric magnetized plasma waveguide and their excitation by an annular relativistic rotating electron beam," *Plasma physics and controlled fusion*, vol. 48, pp. 1105-1123, 2006.
- [3] B. Jazi, M. Nejati, and B. Shokri, "Excitation of THz symmetric TM-modes in a cylindrical metallic waveguide with an axial magnetized degenerate plasma rod by an electron beam," *Physics Letters*, vol. A 370, pp. 319-330, 2007.
- [4] H. Tashakori, A. Niknam, M. Nejati, and B. Jazi, "Generation and amplification of terahertz electromagnetic waves in a plasma waveguide with a dielectric rod and an annular degenerate plasma," *Waves in Random and Complex Media*, vol. 20, pp. 472-490, 2010.
- [5] B. Shokri and B. Jazi, "Time growth rate of symmetric TM mode of a rod dielectric Cerenkov plasma maser," *Physics of plasmas*, vol. 12, pp. 033-104, 2005.
- [6] X. Hong-Quan and L. Pu-Kun, "Theoretical analysis of a relativistic travelling wave tube filled with plasma," *Chinese Physics*, vol. 16, pp. 766-771, 2007.
- [7] M. Nejati and B. Shokri, "The Rod Degenerate Plasma-Rippled-Wall Waveguide and Its Excitation Relativistic Electron Beam Injection," *IEEE Transactions on Plasma Science*, vol. 40, pp. 3029-3036, 2012.